L.C.M & H.C.F

Factors and Multiples: If a number `m' divides another number `n' exactly, then we say that `m' is a factor of `n' and that `n' is a multiple of `m'.

eg. 3 is a factor of 12 and therefore 12 is a multiple of 3.

Least Common Multiple (L.C.M.)

L.C.M. is the least non-zero number in common multiples of two or more numbers.

Multiple of 6 = 6, 12, 18, 24, 30, ...

Multiple of 8 = 8, 16, 24, 32, 40, ...

Common Multiple of 6 and 8 = 24, 48 ....

Least Common Multiple = 24

Factorisation Method:

Find the L.C.M. of 12, 27 and 40

Factors of 12

= 2x2x3 = 2^2x3

Factors of 27

= 3x3x3 = 3^3

Factors of 40

= 2x2x2x5 = 2^3x5

$$\begin{array}{r|r} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$\therefore$ L.C.M. = $2^3 \times 3^3 \times 5 = 1080$

SHORT CUT METHOD

(Division Method)

Find the L.C.M. of 12, 27, 40

$$\begin{array}{r|l} 2 & 12, 27, 40 \\ \hline 2 & 6, 27, 20 \\ \hline 3 & 3, 27, 10 \\ \hline & 1, \ 9, 10 \end{array}$$

$\therefore$ L.C.M. = 2x2x3x9x10 = 1080

HIGHEST COMMON FACTOR (H.C.F)

The highest common factor of two or more numbers is the greatest number which divides each of them exactly.

eg. Find the H.C.F. of 24 and 56

Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Factors of 56 = 1, 2, 4, 7, 8, 14, 28, 56

Common factors of 24 and 56 are 1, 2, 4, 8

$\therefore$ H.C.F. of 24 and 56 = 8

Factorisation Method: H.C.F. can be found by resolving the given numbers into prime factors and then taking the product of least powers of all common factors, that occur in these numbers.

Eg. Find H.C.F. of 48, 108, 140

Factors of 48

= 2x2x2x2x3

= 2^4x3

Factors of 108

= 2x2x3x3x3 = 2^2x3^3

$$\begin{array}{r|r} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array} \qquad \begin{array}{r|r} 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \qquad \begin{array}{r|r} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

Factors of 140

= 2x2x5x7 = 2^2 x 5 x 7

H.C.F. = 2^2 = 4

Division Method

Find the H.C.F. of 48, 108, 140

$$\begin{array}{r|l} 2 & 48, 108, \ 140 \\ \hline 2 & 24, 54, \ 70 \\ \hline & 12, 27, \ 35 \end{array}$$

H.C.F.=2x2= 4

QUICKER & SHORT CUT METHOD

Find the H.C.F. of 777 and 1147

```
777) 1147 (1
      777
     ----
     370) 777(2
          740
          ----
           37) 370 (10
               370
               ---
                 0
```

H.C.F. of 777 and 1147 is 37

* The product of two given numbers is equal to the product of their H.C.F. and L.C.M.

L.C.M. of two numbers

$$= \frac{\text{Product of numbers}}{\text{H.C.F. of numbers}}$$

L.C.M. of given fractions

$$= \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

H.C.F of given fractions

$$= \frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$$

- ♦ The L.C.M of a given set of numbers would be either the highest or higher than the highest of the given numbers.
- ♦ The H.C.F. of a given set of numbers would be either the lowest or lower than the lowest.

Solved Examples

1. Find the L.C.M. of 125,64,8 and 3.

Ans : Given numbers are 5^3, 2^6, 2^3 and 3

$\therefore$ L.C.M. $5^3 x 2^6 x 3$ = 24,000

2. Find the L.C.M. of $\frac{1}{3}, \frac{5}{6}, \frac{5}{9}, \frac{10}{27}$?

Ans: L.C.M. of fractions

$$= \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

L.C.M. of 1, 5 and 10 is 10

H.C.F of 3, 6, 9 and 27 is 3

L.C.M. of given fractions = $\frac{10}{3}$

3. Find the H.C.F. of $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}$

Ans: H.C.F. of fractions

$$= \frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$$

H.C.F. of 1, 3, 5, 7 and 9 is 1

L.C.M of 2, 4, 6, 8 and 10 is 120

H.C.F. of given fractions = $\frac{1}{120}$

4. The L.C.M. of two number is 2310. Their H.C.F. is 30. If one number is 210, the other is:

Ans: The other number

$$= \frac{\text{L.C.M. x H.C.F.}}{\text{given number}} = \frac{2310 \times 30}{210} = 330$$

5. The H.C.F. and L.C.M. of two numbers are 44 and 264 respectively. If the first number is divided by 2, the quotient is 44, The other number is

Ans: First number = 2x44 = 88

Second number = $\frac{44x264}{88} = 132$

6. The least square number which is divisible by 6, 8 and 15 is:

Ans: The least number divisible by 6, 8 and 15 is their L.C.M. which is 120

Now 120 = 2x2x2x3x5

To make it a perfect square, it must be multiplied by 2x3x5

$\therefore$ Required Number=120x2x3x5=3600

7. The least number of square tiles required to pave the ceiling of a room 15m 17cm long and 9m. 2cm broad is:

Ans: Size of largest square tile

= H.C.F. of 1517 cm and 902 cm

= 41 cm.

$\therefore$ Least number of tiles required

$$= \frac{\text{Area of the room}}{\text{Area of one tile}}$$

$$= \frac{1517 \text{ x } 902}{41 \text{ x } 41} = 814$$

8. Find the least number which when divided separately by 15, 20, 36 and 48 leaves 3 as remainder in each case.

Ans : Required number

= L.C.M. of (15,20,36 and 48) +3

= 720 + 3 = 723

9. Find the greatest number that will divide 197 and 269 and leaves 5 as remainder in each case.

Required number = H.C.F. of [(197-5) and (269-5)]

= H.C.F. of (192 and 264) = 8

12. Five bells begin to toll together and toll respectively at intervals of 6,7,8,9 and 12 seconds. How many times they will toll together in one hour, excluding the one at the start?

Ans: L.C.M. of 6,7,8,9 and 12

= 2x2x3x7x2x3 = 504

ie, The bells will toll together after each 504 seconds. In one hour, they will toll together

$$= \frac{60 \text{x} 60}{504} = 7 \text{ times}$$

PRACTICE TEST

1. Find the L.C.M of 12, 15, 18 and 27.
   a) 1,080 b) 540
   c) 270 d) 760

2. Find the H.C.F. of 72, 48 and 30.
   a) 30 b) 12 c) 6 d) 3

3. Find the L.C.M. of $2^2 \text{x} 3^3 \text{x} 5^3$ and $2^3 \text{x} 3^2 \text{x} 5$.
   a) 27,000 b) 180
   c) 36 d) 13,500

4. Find the L.C.M. of $\frac{2}{5}, \frac{3}{10}$ and $\frac{4}{15}$
   a) $\frac{1}{30}$ b) $2\frac{2}{5}$
   c) $\frac{24}{750}$ d) $\frac{2}{5}$

5. Find the H.C.F. of $\frac{4}{5}, \frac{3}{10}$ and $\frac{7}{15}$
   a) $\frac{1}{5}$ b) $\frac{84}{5}$
   c) $\frac{84}{30}$ d) $\frac{1}{30}$

6. If the L.C.M of x and y is z, their H.C.F. is.
   a) $\frac{xy}{z}$ b) xyz
   c) $\frac{x+y}{z}$ d) $\frac{z}{xy}$

7. H.C.F of two numbers is 24 and their L.C.M is 1080. If one of the numbers is 120, find the other.
   a) 216 b) 532 c) 108 d) 820

8. L.C.M. of 2.5, 0.5 and 0.175 = ?
   a) 2.5 b) 0.5
   c) 0.175 d) 17.5

9. H.C.F. of two numbers is 24 and their L.C.M is 1344. If the difference between the numbers is 80, their sum is:
   a) 368 b) 356
   c) 332 c) 304

10. Find the greatest number which can divide 1354, 1866 and 2762 leaving the same remainder 10 in each case.
    a) 64 b) 124 c) 156 d) 260

11. Find the least perfect square which is di-

visible by 3, 4, 5, 6 and 8.

a) 2500 b) 1200

c) 3600 d) 900

12. The least number which when divided by 15, 27, 35 and 42 leaves in each case a remainder 7 is:

a) 1897 b) 1987

c) 1883 d) 2007

13. Two containers contain 60 and 165 litres of milk respectively. Find the maximum capacity of a container which can measure the milk in each container an exact number of times (in litres)

a) 15 b) 3 c) 5 d) 10

14. Two baskets contain 195 and 250 bananas respectively, which are distributed in equal number among children. Find the largest number of bananas that can be given, so that 3 bananas are left over from the first basket and 2 from the second.

a) 4 b) 18 c) 8 d) 6

Qn: (15- 18) :- Write in ascending order

15. $\frac{1}{2}, \frac{2}{5}, \frac{3}{4}, \frac{3}{2}$

a) $\frac{2}{5}, \frac{1}{2}, \frac{3}{4}, \frac{3}{2}$ b) $\frac{3}{4}, \frac{1}{2}, \frac{2}{5}, \frac{3}{2}$

c) $\frac{3}{2}, \frac{3}{4}, \frac{1}{2}, \frac{2}{5}$ d) $\frac{3}{2}, \frac{1}{2}, \frac{2}{5}, \frac{3}{4}$

16. $\frac{5}{3}, \frac{11}{9}, \frac{5}{6}, \frac{7}{12}$

a) $\frac{11}{9}, \frac{7}{12}, \frac{5}{3}, \frac{5}{6}$ b) $\frac{7}{12}, \frac{5}{6}, \frac{11}{9}, \frac{5}{3}$

c) $\frac{5}{6}, \frac{7}{12}, \frac{11}{9}, \frac{5}{3}$ d) $\frac{5}{3}, \frac{11}{9}, \frac{5}{6}, \frac{7}{12}$

17. $\frac{5}{6}, \frac{7}{8}, \frac{3}{4}, \frac{1}{3}$

a) $\frac{7}{8}, \frac{1}{3}, \frac{3}{4}, \frac{5}{6}$ b) $\frac{5}{6}, \frac{7}{8}, \frac{3}{4}, \frac{1}{3}$

c) $\frac{3}{4}, \frac{7}{8}, \frac{1}{3}, \frac{3}{4}$ d) $\frac{1}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}$

Qn: 18- 20 Write in descending order

18. $\frac{1}{3}, \frac{2}{5}, \frac{3}{4}, \frac{1}{6}$

a) $\frac{1}{3}, \frac{2}{5}, \frac{3}{4}, \frac{1}{6}$ b) $\frac{1}{6}, \frac{2}{5}, \frac{1}{3}, \frac{3}{4}$

c) $\frac{2}{5}, \frac{3}{4}, \frac{1}{3}, \frac{1}{6}$ d) $\frac{3}{4}, \frac{2}{5}, \frac{1}{3}, \frac{1}{6}$

19. $\frac{5}{6}, \frac{7}{8}, \frac{11}{12}, \frac{3}{10}$

a) $\frac{5}{6}, \frac{7}{8}, \frac{11}{12}, \frac{3}{10}$ b) $\frac{7}{8}, \frac{5}{6}, \frac{11}{12}, \frac{3}{10}$

c) $\frac{11}{12}, \frac{7}{8}, \frac{5}{6}, \frac{3}{10}$ d) $\frac{7}{8}, \frac{5}{6}, \frac{11}{12}, \frac{3}{10}$

20. $\frac{5}{3}, \frac{11}{9}, \frac{5}{6}, \frac{7}{12}$

a) $\frac{5}{3}, \frac{11}{9}, \frac{5}{6}, \frac{7}{12}$ b) $\frac{11}{9}, \frac{5}{3}, \frac{7}{12}, \frac{5}{6}$

c) $\frac{5}{3}, \frac{11}{9}, \frac{5}{6}, \frac{7}{12}$ d) $\frac{11}{9}, \frac{5}{6}, \frac{5}{3}, \frac{7}{12}$

Qn 21-23 Find the greatest of the given fractions

21. $\frac{2}{3}, \frac{4}{15}, \frac{3}{5}, \frac{3}{4}$

a) $\frac{4}{15}$ b) $\frac{3}{4}$

c) $\frac{3}{5}$ d) $\frac{2}{3}$

22. $\frac{5}{8}, \frac{6}{11}, \frac{13}{22}, \frac{9}{13}$

a) $\frac{5}{8}$ b) $\frac{6}{11}$

c) $\frac{13}{22}$ d) $\frac{9}{13}$

23. $\frac{3}{4},\frac{5}{7},\frac{2}{3},\frac{8}{11}$

a) $\frac{3}{4}$ b) $\frac{5}{7}$

c) $\frac{2}{3}$ d) $\frac{8}{11}$

Qn: (24 - 26) Find the smallest of the given fraction.

24. $\frac{2}{3},\frac{5}{7},\frac{9}{13},\frac{9}{14},\frac{7}{4}$

a) $\frac{9}{14}$ b) $\frac{2}{3}$

c) $\frac{7}{4}$ d) $\frac{5}{7}$

25. $\frac{11}{14},\frac{14}{17},\frac{17}{20},\frac{23}{26},\frac{29}{32}$

a) $\frac{29}{32}$ b) $\frac{11}{14}$

c) $\frac{17}{20}$ d) $\frac{14}{17}$

26. $\frac{5}{6},\frac{3}{4},\frac{5}{8},\frac{6}{7}$

a) $\frac{3}{4}$ b) $\frac{6}{7}$

c) $\frac{5}{8}$ d) $\frac{5}{6}$

27. A heap of stones can be made in groups of 21 but when made up into groups of 16, 20, 25 and 45 there are 3 stones left in each case, The number of stones in the heap is
a) 3600 b) 3603
c) 7200 d) 7203

28. Three measuring rods are 64cm, 80cm and 96 cm in length. The least length of cloth (in metres) that can be measured exact number of times using any of the three rods is
a) 0.96m b) 9.6m
c) 96m d) 960m

29. The largest number, which exactly divides the product of any three consecutive integers is
a) 2 b) 3 c) 6 d) 12

30. The L.C.M. of two numbers is 63 and their H.C.F. is 9. If one of the numbers is 27, the other number will be
a) 9 b) 21 c) 17 d) 189

31. The HCF of two numbers is 32 and their product is 10240. Find their L.C.M?
a) 640 b) 320 c) 324 d) 230

32. A gardener had a number of shrubs to plant in rows. At first he tried to plant 8, then 12 and then 16 in a row but he had always 3 shrubs left with him. On trying 7 he had none left. Find the total number of shrubs.
a) 147 b) 150 c) 137 d) 154

33. Six bells commencing tolling together and toll at intervals of 2,4,6,8,10 and 12 seconds respectively. In 30 minutes, how many times do they toll together.
a) 17 b) 15 c) 16 d) 20

34. In a seminar the number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively. Find the minimum number of rooms required, where in each room the same number of participants are to be seated; and all of them being in the same subject.
a) 20 b) 22 c) 25 d) 21

ANSWERS TO PRACTICE TEST - 6

1. (b)	2. (c)	3. (a)	4. (b)	5. (d)
6. (a)	7. (a)	8. (d)	9. (a)	10. (a)
11. (c)	12. (a)	13. (a)	14. (c)	15. (a)
16. (b)	17. (d)	18. (d)	19. (c)	20. (a)
21. (b)	22.(d)	23. (a)	24. (a)	25. (b)
26. (c)	27. (d)	28.(b)	29. (c)	30. (b)
31. (b)	32. (a)	33. (b)	34. (d)	