

Square & Square Roots

Square : If a number is multiplied by itself then the product is the square of the number. Thus the square of 5 is $5 \times 5 = 25$

$$\text{eg. } \left(\frac{1}{2}\right)^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$$

Square root: The square root of a number is one of two equal factors which is multiplied together gives that number.

$$\text{eg. } \sqrt{121} = \sqrt{11 \times 11} = 11$$

$$\sqrt{10000} = \sqrt{100 \times 100} = 100$$

Finding Square root by means of factorisation

When a given number is a perfect square, we resolve it into prime factors and take the product of prime factors, choosing one out of each pair.

eg. Find the square root of 1156

Factors of 1156 is $2 \times 2 \times 17 \times 17$

$$1156 = 2 \times 2 \times 17 \times 17 = 2^2 \times 17^2$$

$$\sqrt{1156} = \sqrt{2^2 \times 17^2} = 2 \times 17 = 34$$

General method to find the square root

In the given number mark off the digits in pairs, from right and then find the square root as shown in the example below.

eg. 1. Find the square root of 219961

$$\begin{array}{r} 469 \\ 4 \overline{) 21,99,61} \\ \underline{16} \\ 86 \\ \underline{599} \\ 829 \\ \underline{8361} \\ 0 \end{array}$$

$$\therefore \sqrt{219961} = 469$$

2. Find the square root of 59.1361

$$\begin{array}{r} 7.69 \\ 7 \overline{) 59.1361} \\ \underline{49} \\ 146 \\ \underline{1013} \\ 1529 \\ \underline{13761} \\ 0 \end{array}$$

$$\therefore \sqrt{59.1361} = 7.69$$

Properties of a perfect square

- ◆ No perfect square ends with 2,3,7,8
- ◆ No perfect square ends with an odd number of zeros.
- ◆ The perfect square consisting of (n-1) or n digits will have $\frac{n}{2}$ digits in their root
- ◆ The square of a number other than unity is either a multiple of 4 or exceeds a multiple of 4 by 1.

CUBE ROOT

The cube root of a number is one of three equal factors which if multiplied gives that number. Cube root of a number can be found out by using the following steps.

1. Write down all the prime factors of the given numbers.
2. Write the prime factors in the index notation, ie, in a^n form.
3. Divide the index by 3, then the result will be the cube root of the given number.

eg. 1. Find the cube root of 512

$$512 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$$

$$\sqrt[3]{512} = (2^9)^{1/3} = 2^3 = 8$$

2. Find the cube root of 0.000027

$$3. \sqrt[3]{0.000027} = \left(\frac{27}{1000000}\right)^{1/3} = \frac{(3^3)^{1/3}}{(100^3)^{1/3}}$$

$$= \frac{3}{100} = 0.03$$

Learn by heart the following square roots

SQUARES AND CUBES

No.	Square	Cube	No	Square
1	1	1	16	256
2	4	8	17	289
3	9	27	18	324
4	16	64	19	361
5	25	125	20	400
6	36	216	21	441
7	49	343	22	484
8	64	512	23	529
9	81	729	24	576
10	100	1000	25	625
11	121	1331	26	676
12	144	1728	27	729
13	169	2197	28	784
14	196	2744	29	841
15	225	3375	30	900

$\sqrt{2} = 1.41421$	$\sqrt{6} = 2.44949$
$\sqrt{3} = 1.73205$	$\sqrt{7} = 2.64575$
$\sqrt{5} = 2.2360$	$\sqrt{10} = 3.1622$

PRACTICE TEST

1. If $x = \sqrt{3018 + \sqrt{36} + \sqrt{169}}$, the value of x is

- a) 44 b) 55 c) 69 d) 43

2. If $x = \sqrt{\frac{169 \times 0.9}{13 \times 0.13}}$, the value of x is

- a) 3 b) 13 c) 39 d) 0.39

3. If $\sqrt{\frac{x}{49}} = \frac{4}{7}$, the value of x is

- a) 7 b) 49 c) 16 d) 4

4. If $x = \frac{\sqrt{196}}{7} + \frac{18}{\sqrt{324}} + \frac{65}{\sqrt{169}}$ the value of x is

- a) 12 b) 10 c) 8 d) 15

5. $\frac{\sqrt{484}}{11} \times \frac{\sqrt{324}}{9} \times 0.4 = ?$

- a) $\frac{5}{8}$ b) $\frac{8}{5}$ c) $\frac{9}{5}$ d) $\frac{10}{5}$

6. If $\frac{169}{\sqrt{x}} = 169$, the value of x is

- a) 100 b) 1000
c) 10000 d) 1,00,000

7. $\frac{\sqrt{144}}{6} + \frac{\sqrt{324}}{6} \times \frac{104}{\sqrt{169}} = ?$

- a) 26 b) 14
c) 101.82 d) 36

8. $8\sqrt{?} + 44 = 25\%$ of 400

- a) 3 b) 36 c) 49 d) 16

9. If $x = \sqrt{\frac{81}{0.09}}$, the value of x is

- a) 3 b) 30 c) 300 d) 0.3

10. The largest of four digit numbers which is a perfect square is

- a) 9801 b) 9904
c) 9804 d) 9809

11. A certain number of people collected Rs. 125. If each person contributed as many five paise as they are in number, the number of person were

- a) 25 b) 50 c) 100 d) 125

12. A gardener plants an orchard with 5776 trees. In each row there were as many trees as the number of rows. Find the number of rows.

- a) 76 b) 96 c) 66 d) 186

13. Each student in a class contributed as many rupees as the number of students in the class for a picnic. The school contributed Rs. 150 per teacher who led the trip. If the total amount collected was Rs. 1350 and the number of teachers who led the trip was 3, how many students were there in that class?

- a) 36 b) 35 c) 34 d) 30

14. Some persons contributed Rs. 1089. Each person gave as many rupees as they are in number. Find their number.

- a) 33 b) 66 c) 45 d) 23

15. If $\sqrt{1 + \frac{x}{144}} = \frac{13}{12}$ then x is equal to.

- a) 0 b) 12 c) 13 d) 25

16. $\left(\sqrt{\frac{169}{225}} - \sqrt{\frac{4}{9}}\right) \div \sqrt{\frac{81}{121}} = ?$

- a) $\frac{9}{55}$ b) $\frac{11}{45}$ c) $4\frac{1}{11}$ d) $\frac{45}{11}$

17. $\frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} - \sqrt{5}}$ is equal to

- a) $6 + \sqrt{35}$ b) $6 - \sqrt{35}$
c) 2 d) 1

18. If $\sqrt{0.04 \times 0.4 \times a} = 0.004 \times 0.4 \times \sqrt{b}$,

then $\frac{a}{b}$ is

- a) 16×10^{-4} b) 16×10^{-3}
c) 16×10^{-5} d) 16×10^{-6}

19. The value of $\left(\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{2+\sqrt{3}}\right)$ is

- (a) -1 b) 0
c) 1 d) $\frac{1}{3} \left(\frac{1}{\sqrt{2} + \sqrt{3}}\right)$

20. The value of $\sqrt{400 + \sqrt{1681}}$ is

- a) 41 b) 21 c) 31 d) 51

21. if $\sqrt{5} = 2.24$ and $\sqrt{6} = 2.45$, the

value of $\sqrt{\frac{2}{3}} + \sqrt{\frac{5}{6}}$ is

- a) 1.37 b) 1.57 c) 1.73 d) 1.75

22. If $\sqrt{15625} = 125$ then the value of

$\sqrt{15625} + \sqrt{156.25} + \sqrt{1.5625}$ is

- a) 1.3875 b) 13.875
c) 138.75 d) 156.25

23. If $\sqrt{4096} = 64$ to then the value of

$\sqrt{40.96} + \sqrt{0.4096} + \sqrt{0.004096} + \sqrt{0.0004096}$ is

- a) 7.09 b) 7.1014
c) 7.1104 d) 7.12

24. The expression

$2 + \sqrt{2} + \frac{1}{2 + \sqrt{2}} + \frac{1}{\sqrt{2} - 2}$ equals

- a) 2 b) $2\sqrt{2}$
c) $2 - \sqrt{2}$ d) $2 + \sqrt{2}$

ANSWERS TO PRACTICE TEST

1. (b) 2. (a) 3. (c) 4. (c) 5. (b) 6. (c) 7. (a) 8. (c)
9. (b) 10. (a) 11. (b) 12. (a) 13. (d) 14. (a) 15. (d) 16. (b)
17. (a) 18. (c) 19. (c) 20. (b) 21. (c) 22. (c) 23. (c) 24. (a)

Laws of Exponents

A long product axaxa ...m factors can be expressed, in short by notation a^m , where 'a' is called the base and 'm' the power (or index or exponent) Thus axaxa.... 10 factors = a^{10} .

Definition a^m : If a is a natural number, then a^m stands for the product of 'm' factors each equal to 'a'.

Property 1: $a^m \times a^n = a^{m+n}$

(eg. $a^3 \times a^4 = a^{3+4} = a^7$)

Property 2 : $a^n = a^{m-n}$

(eg. $a^5 = a^{5-3} = a^2$)

Property 3 : $\frac{a^m}{a^n} = a^{m-n}$

(eg. $\frac{a^6}{a^2} = a^4$)

Meaning of a^0 : Any non-zero number raised to zero power is equal to 1.

ie. $a^0 = 1$ where $a \neq 0$ eg. $5^0 = 1$.

Negative Index : If $a \neq 0$ then $a^{-m} = \frac{1}{a^m}$

(eg. $a^{-3} = \frac{1}{a^3}$)

Fractional Index : $a^{1/n} = \sqrt[n]{a}$

eg. $a^{1/2} = \sqrt{a}$, $a^{2/3} = \sqrt[3]{a^2}$

If 'a' and 'b' are non-zero integers, 'm' and 'n' are rational numbers, then

i). $a^m \times a^n = a^{m+n}$

ii). $\frac{a^m}{a^n} = a^{m-n}$

iii). $(a^m)^n = a^{mn}$

iv). $(ab)^m = a^m \cdot b^m$

v). $a^0 = 1$

vi). $a^{-m} = \frac{1}{a^m}$

SOLVED EXAMPLES :

1. $8^{1/3} \times 16^{1/4} \times 2^{-2} = ?$

$$(2^3)^{1/3} \times (2^4)^{1/4} \times 2^{-2}$$

$$2^1 \times 2^1 \times 2^{-2} = 2^{1+1-2} = 2^0 = 1$$

2. $(2^2)^3 \times (2^3)^2 \div 2^6 = ?$

$$\frac{2^6 \times 2^6}{2^6} = 2^6 = 64$$

3. $(3^2)^2 \times (3^2)^4 \div (27)^4$

$$\frac{3^4 \times 3^8}{(3^3)^4} = \frac{3^4 \times 3^8}{3^{12}} = \frac{3^{12}}{3^{12}} = 1$$

4. Evaluate

$$3^0 + 3^{-2} + (27)^{2/3} + \sqrt[3]{2^6} + \left(\frac{9}{25}\right)^{-1/2}$$

$$= 1 + \frac{1}{3^2} + (3^3)^{2/3} + (2^6)^{1/3} + \left[\left(\frac{3}{5}\right)^2\right]^{-1/2}$$

$$= 1 + \frac{1}{9} + 3^2 + 2^2 + \frac{5}{3}$$

$$= 1 + \frac{1}{9} + 9 + 4 + \frac{5}{3} = 14 + 1\frac{7}{9} = 15\frac{7}{9}$$

5. If $5^a = 3125$, find the value of 5^{a-3}

$$5^a = 3125 \Rightarrow 5^a = 5^5$$

$$\Rightarrow a = 5$$

$$\therefore 5^{a-3} = 5^{5-3} = 5^2 = 25$$

6. If m and n are whole numbers such that $m^n = 121$, find the value of $(m-1)^{n+1}$

$$m^n = 121 = 11^2 \Rightarrow m = 11, n = 2$$

$$\therefore (m-1)^{n+1} = (11-1)^{2+1} = 10^3 = 1000$$

PRACTICE TEST

- 25^{25} is divided by 24, the remainder is
a) 23 b) 22 c) 1 d) 2
- $(64)^{-1/2} - (-32)^{-4/5}$ equals
a) $\frac{1}{16}$ b) $\frac{1}{8}$ c) $\frac{3}{16}$ d) 2
- If $\sqrt{2^n} = 64$, the value of n is
a) 2 b) 4 c) 6 d) 12
- If $1.125 \times 10^k = 0.001125$, the value of k is
a) -4 b) -3 c) -2 d) -1
- The unit's digit in the product of $(2467)^{153} \times (341)^{72} \times (225)^{721}$ is
a) 1 b) 3 c) 5 d) 7
- If $2^{2x-1} = \frac{1}{8^{x-3}}$, the value of x is
a) 0 b) -1 c) -2 d) 2
- The value of $\frac{3^{12+n} \times 9^{2n-7}}{3^{5n}}$ is
a) $\frac{1}{3}$ b) $\frac{9}{13}$ c) $\frac{1}{9}$ d) $\frac{2}{3}$
- If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$, x is equal to
a) 1 b) $\frac{1}{2}$ c) $\frac{7}{2}$ d) 2
- A boy was asked to write the value of $2^5 \times 9^2$. He wrote it as 2592. The difference between the obtained and the actual value is
a) $2^1 \times 9^2$ b) $2^2 \times 9^3$
c) $2^3 \times 9^4$ d) Zero
- $5^{1/4} \times (125)^{0.25}$ is equal to
a) $\sqrt{5}$ b) 25 c) 5 d) $5\sqrt{5}$
- If $a^3 b = ab^3 c = 180$ and $a, b,$ and c are positive integers, the value of c is
a) 10 b) 15 c) 25
d) data inadequate
- $(4)^{0.5} \times (0.5)^4$ is equal to
a) 1 b) 4 c) $\frac{1}{8}$ d) $\frac{1}{32}$
- If $2^{x-1} + 2^{x+1} = 1280$, then the value of x is
a) 7 b) 8 c) 9 d) 10
- The simplification of $\left(\frac{-1}{125}\right)^{-2/3}$ gives
a) $\frac{1}{25}$ b) $\frac{-1}{25}$
c) 25 d) -25
- The units digit in $(2469)^{153}$ will be
a) 1 b) 3 c) 7 d) 9
- $(243)^{0.12} \times (243)^{0.08}$ equals
a) 3 b) 9 c) 12 d) 27
- The value of $4^7 \div 16^4 \times \sqrt{16}$ is equal to:
a) $\frac{1}{16}$ b) $\frac{1}{4}$ c) 4 d) 1

ANSWERS TO PRACTICE TEST

1. (c) 2. (a) 3. (d) 4. (b) 5. (c) 6. (d) 7. (c) 8. (d)
9. (d) 10. (c) 11. (d) 12. (c) 13. (c) 14. (c) 15. (d) 16. (a)
17. (d)